Hysteresis operators II

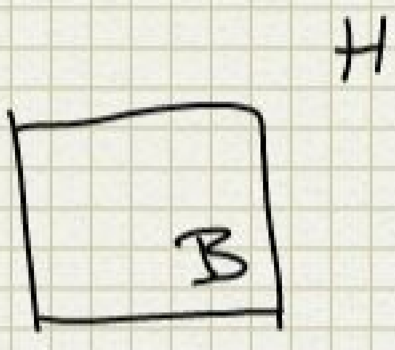
3.5.2013

Outline
- Historical notes
- Play
- Stop
- Relay

I History

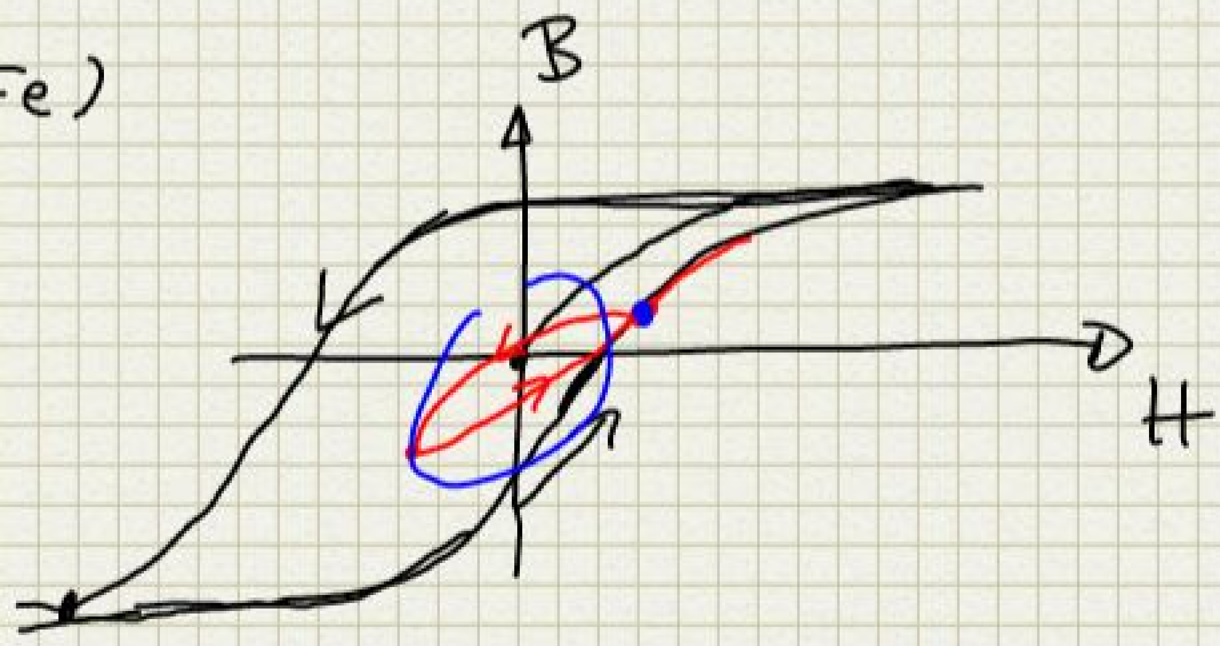
- Ferromagnetism

vacuum $\quad B = \mu_0 H$

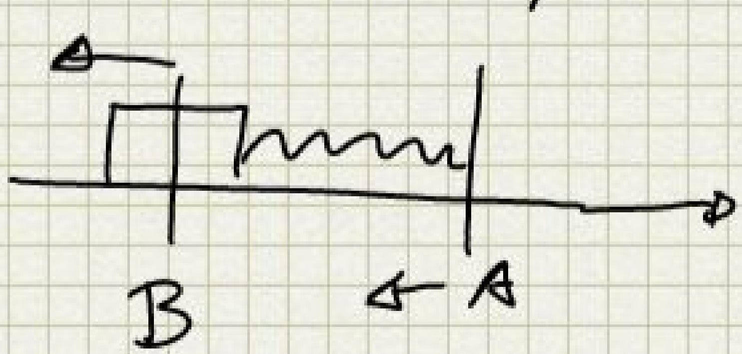


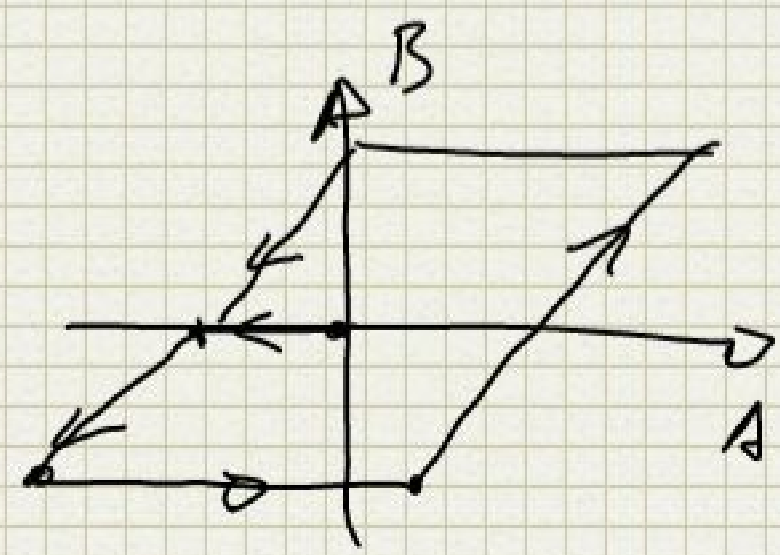
$B = \mu (H + M)$

(Fe)



- Elasto-plasticity





play

- 1983 / 89 Russian group.

94

96 Brokate

98

- Play

$$K = [a, b] \ni 0$$

$$u(t) - w(t) \in K$$

$$\dot{w}(t)\,(u(t) - w(t) - \xi) \geq 0 \quad \forall \xi \in K$$

$$\forall t \in [0, T].$$

Theom: $\forall u \ \exists !\, w$

PROP. play is RI.

Let φ feasible time info.

- $\underline{\mathcal{F}(u \circ \varphi, w^0)(t) = \underline{\mathcal{F}(u, w^0)}(\varphi(t))}$

rhs $\dot{w}(\varphi(t))\,(u(\varphi(t)) - w(\varphi(t)) - \xi) \geq 0 \quad \forall \xi \in K$

$$\dot{w}(\varphi(t)) = \frac{d}{dt}(w \circ \varphi)(t) \cdot \underbrace{\varphi'(t)^{-1}}_{> 0}$$

$$\frac{d}{dt}(w \circ \varphi)(t)\,(\underline{u \circ \varphi(t)} - w \circ \varphi(t) - \xi) \geq 0 \quad \forall \xi$$

lhs $\dot{\tilde{w}}(t)\,(\underline{u \circ \varphi}(t) - \tilde{w}(t) - \xi) \geq 0 \ \forall \xi$

$$\tilde{w} = w \circ \varphi \qquad \square$$

Properties

$$\mathcal{F} : \overbrace{\mathcal{C}[0,T]}^{\mathcal{C}([0,T],\mathbb{R}^n)} \times \underset{\substack{\uparrow \\ X}}{\mathbb{R}} \to \mathcal{C}([0,T])$$

Lipschitz.

- $\forall \, (u, w^0) \in Dom(\mathcal{F}) \subseteq W^{1,1} \times \mathbb{R}$

$$\leadsto \quad \left| \frac{d}{dt} \mathcal{F}(u, w^0) \right| \leq \left| \frac{d}{dt} u \right| \quad \text{a.e. } [0,T]$$

- $$\mathcal{F} : \mathcal{C}[0,T] \times \underset{\substack{\uparrow \\ \text{fix}}}{\mathbb{R}} \to \mathcal{C}\,\underline{[0,T] \cap BV(0,T)}$$

$\mathcal{F}(\cdot, w^0)$ cts.

• Stop

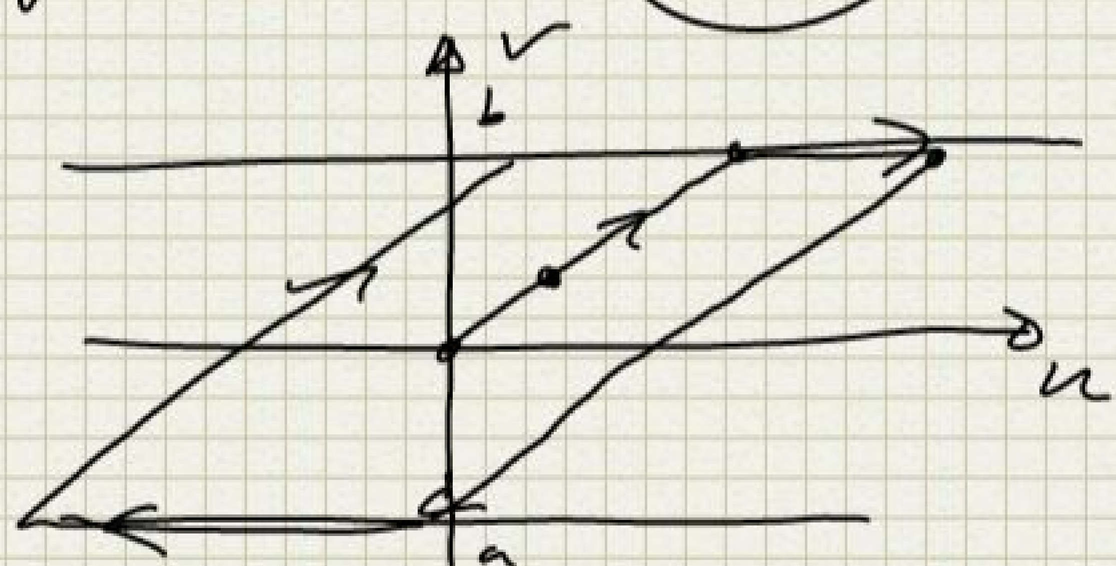
Play $u - w \in K$: $\dot{w}(u - w - \xi) \geq 0 \quad \forall \xi \in K$

write $v = u - w$

Def $v \in K$: $(\dot{u} - \dot{v})(v - \xi) \geq 0 \quad \forall \xi \in K$

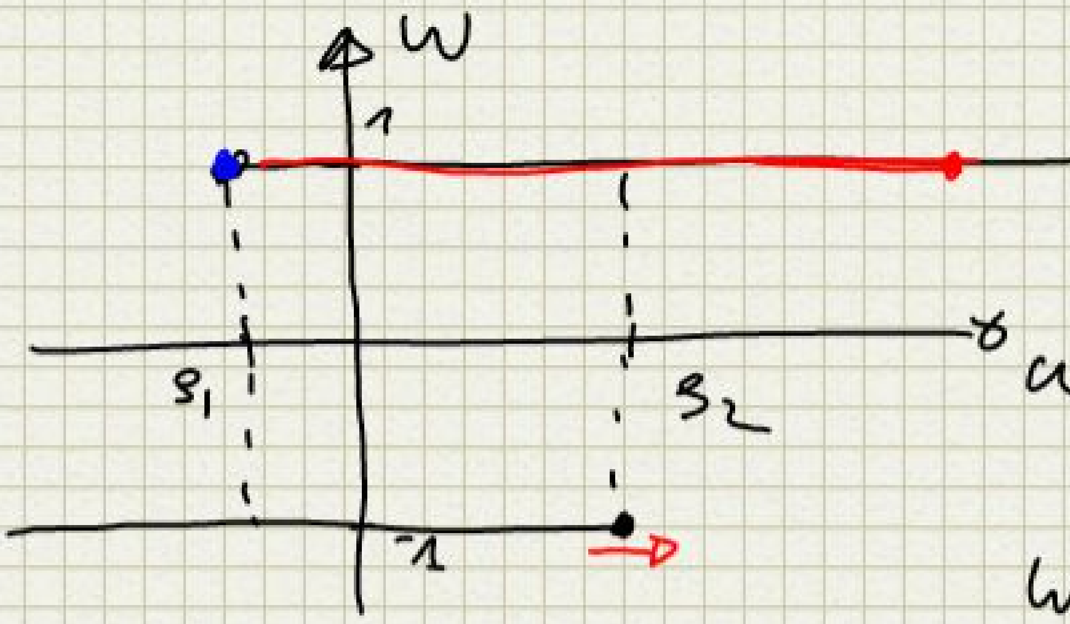
$$\Longleftrightarrow \quad \dot{u} - \dot{v} \in \partial I_K(v) = \begin{cases} 0 & v \in \mathring{K} \\ \mathbb{R}^- & v = a \\ \mathbb{R}^+ & v = b \end{cases}$$

if $v \in \mathring{K}$: $\dot{u} = \dot{v}$

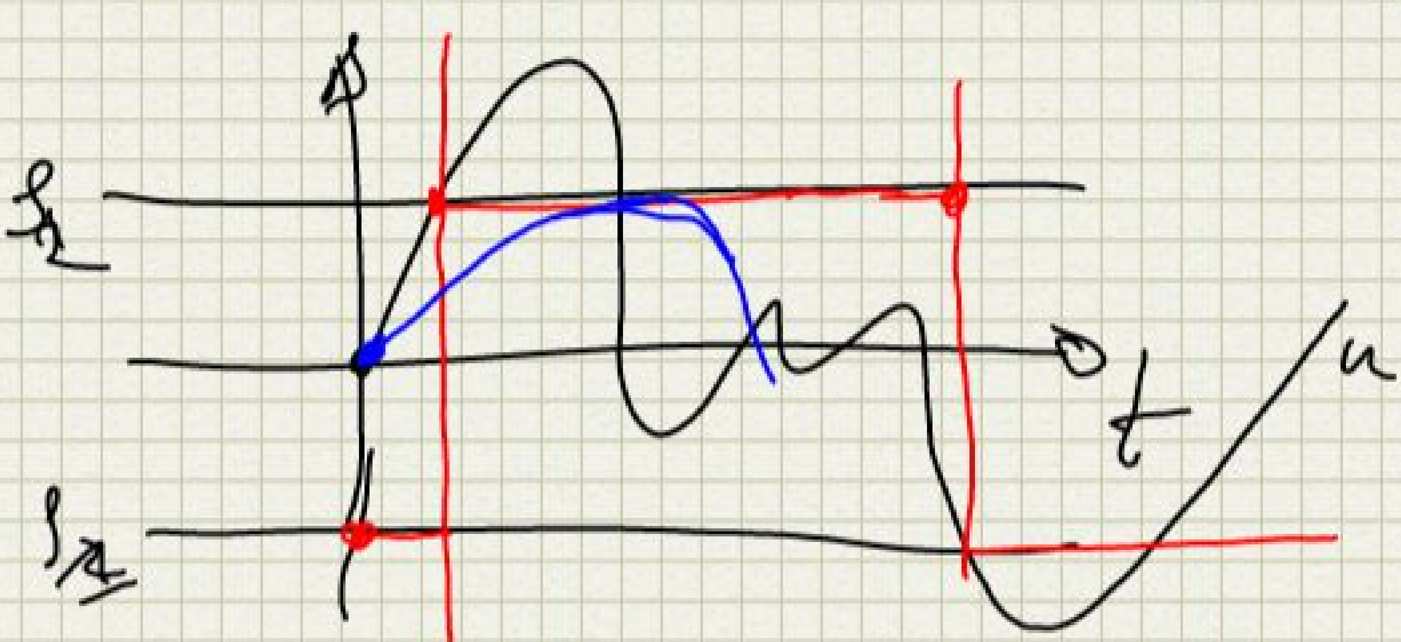


• BWT not BV-regularizing

Relay $\qquad \varrho_1 < \varrho_2$



$w : [0,T] \to \{\pm 1\}$



$$R : \mathcal{C}[0,T] \times \{\pm 1\} \to \mathcal{C}_r[0,T] \cap BV(0,T)$$

$$u(t) := \varrho_2 - (t-1)^2 \qquad [0,2]$$

$$u_\varepsilon^\pm(t) := u(t) \pm \varepsilon$$

u_ε^+ switches in $[0,2]$

u_ε^- no switching

$$u(t) = \lim_{\varepsilon \to 0} u_\varepsilon^\pm(t) \qquad \text{uniformly}$$

but $\bullet \lim_{\varepsilon \downarrow 0} w_\varepsilon^-(t) = -1$

$\cdot \lim_{\varepsilon \downarrow 0} w_\varepsilon^+(t) = \begin{cases} -1 & t<1 \\ \{\pm 1\} & t=1 \\ 1 & t>1 \end{cases}$

- Preisach Operator

$\mathcal{P} = \{(\rho_1, \rho_2) \in \mathbb{R}^2 \,/\, \rho_1 < \rho_2\}$

$\mathcal{R} := \{ \xi : \mathcal{P} \to \{\pm 1\} : \text{Borel} \}$

fix μ finite (signed) Borel measure on $\mathcal{P}$

$\forall \rho \in \mathcal{P}$: Relay R_{ρ_1, ρ_2}

$\mathcal{H}_\mu : \mathcal{C}[0,T] \times \mathcal{R} \to L^\infty(0,T) \cap \mathcal{C}_r([0,T[$

$[\mathcal{H}_\mu(u, \xi)](t) :=$

$= \int_{\mathcal{P}} [R_{\rho_1, \rho_2}(u, \xi_{\rho_1, \rho_2})](t) \, d\mu(\rho_1, \rho_2)$