Hysteresis operators I

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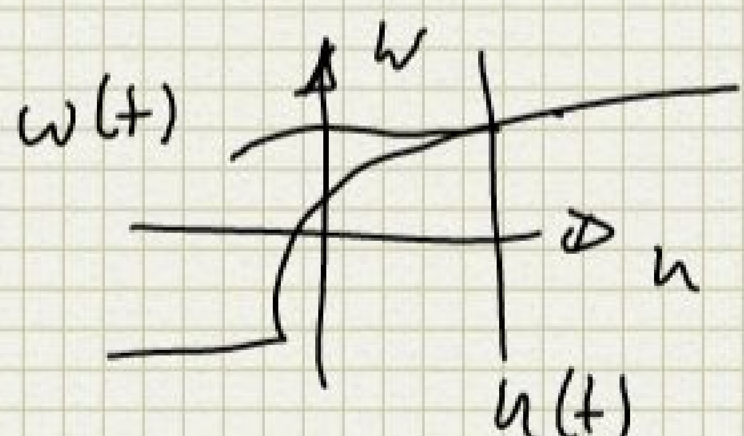
Outline
I What is hysteresis?
II Historical note
III Basics
IV Examples
· Play
· Stop
· Relay

I. "ὑστέρησις" means "to lay behind".

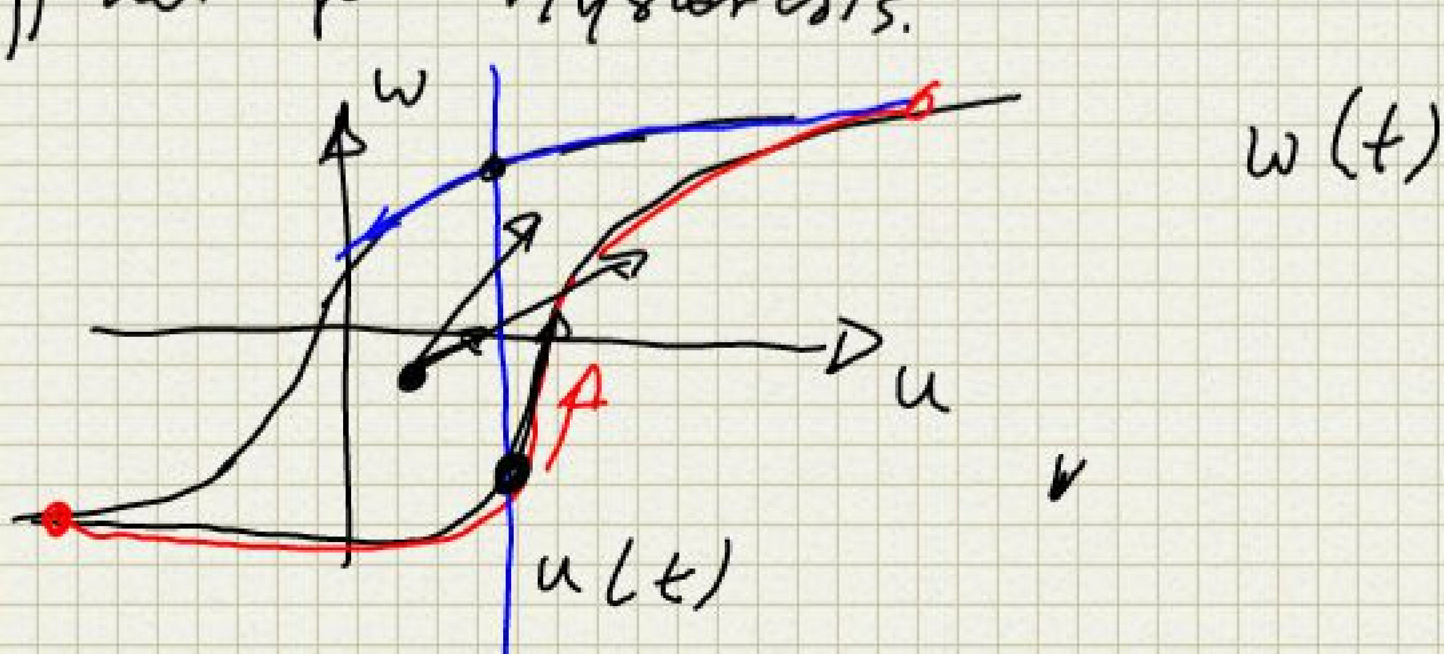
System $(u(t), w(t))$
input output
control state

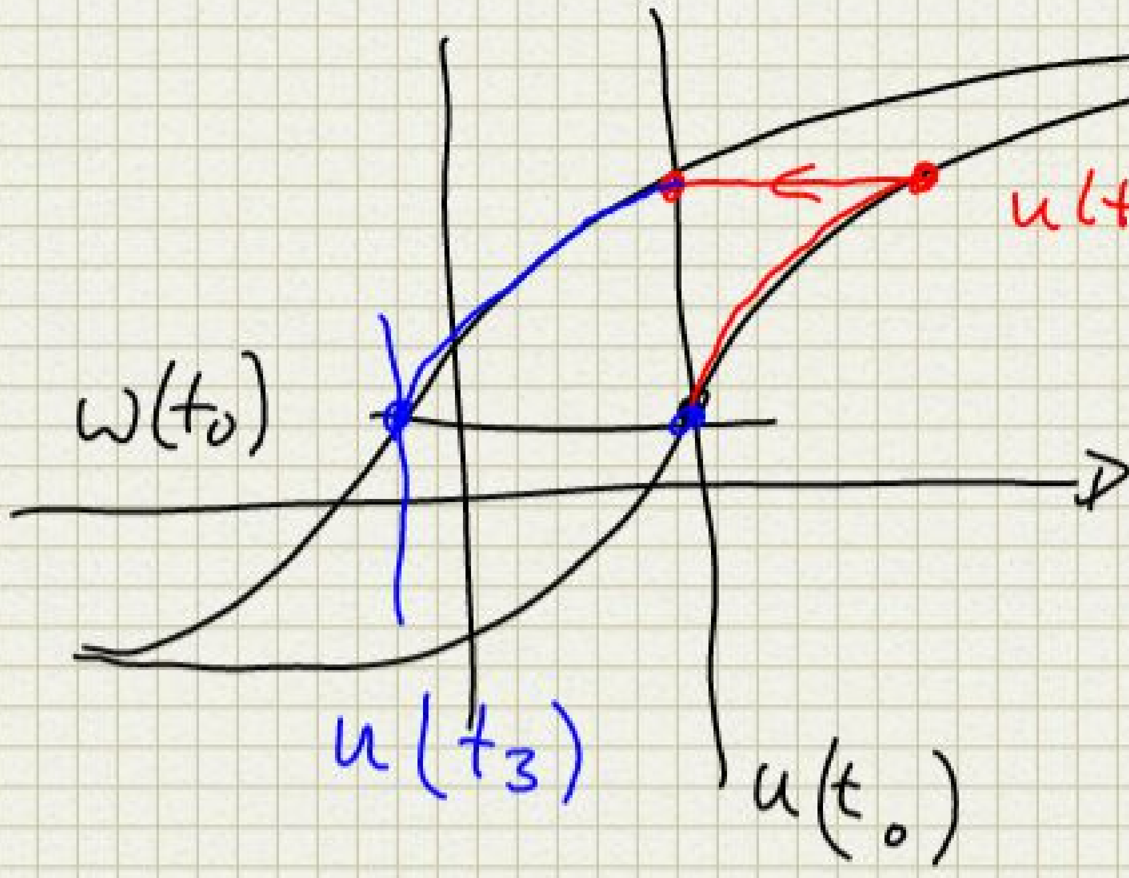
$u \sim w$

· $w(t) = \phi(u(t)) \qquad \phi : \mathbb{R} \to \mathbb{R}$.



different for hysteresis.

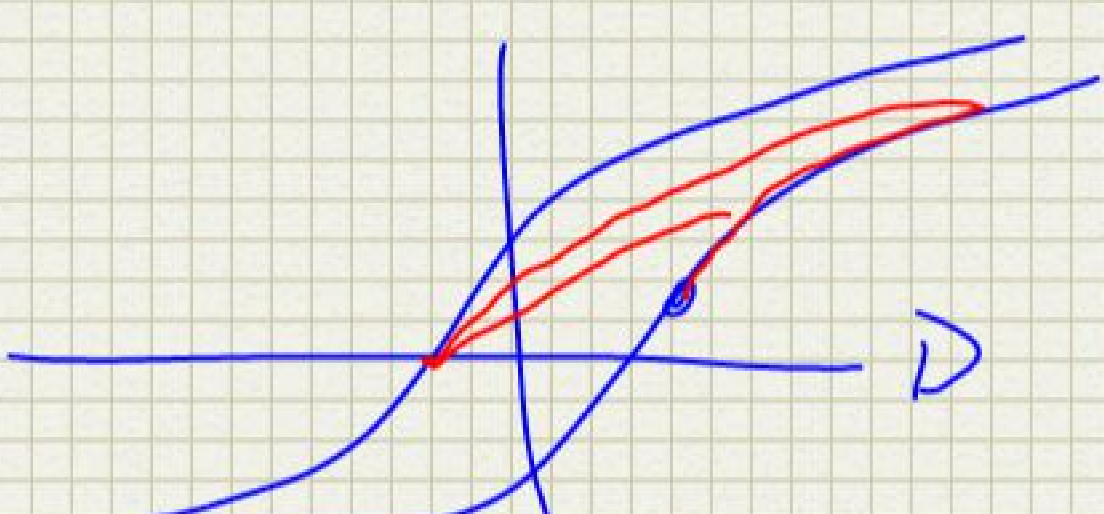




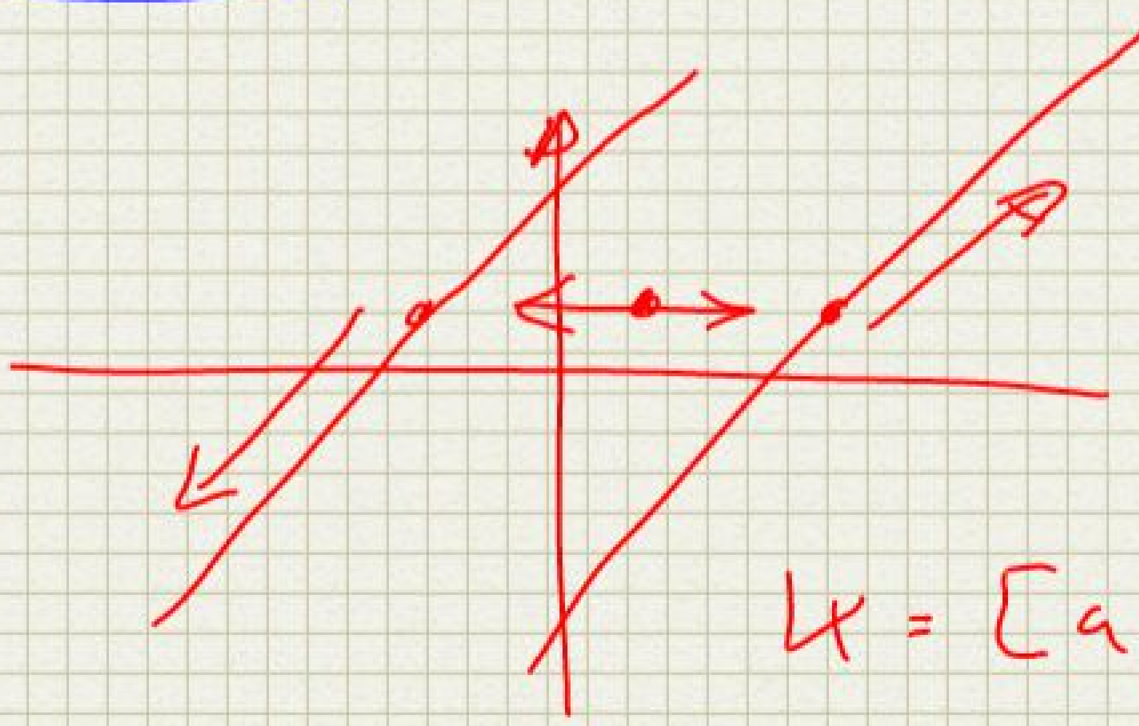
$$\boxed{u(t_2) = u(t_0)}$$

$$\boxed{w(t_0) \neq w(t_2)}$$

$$w(t_0) = w(t_3)$$



Play.



$$K = [a, b] \ni 0$$

$$\dot{w} \in \partial I_K(u-w) = \begin{cases} \emptyset & u-w \notin K \\ 0 & u-w \in \mathring{K} \\ \vdots & \end{cases}$$

$$\dot{w} = 0$$

III Basics

$$\mathcal{F}: \mathrm{Dom}(\mathcal{F}) \subseteq \mathcal{C}[0,T] \times \mathbb{R} \to \mathcal{C}[0,T]$$

$$w(t) := [\mathcal{F}(u, w^0)](t)$$

* **Causality / Volterra property**

  $w(t)$ does not depend on $u|_{(t,T]}$

* **Rate-independence (RI)**

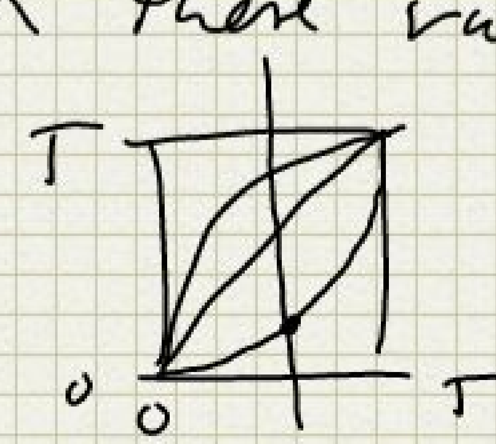
  * adm. time trafo $\varphi: [0,T] \to [0,T]$
    $\varphi(0) = 0$, $\boxed{\varphi(T) = T}$, φ nondecreasing
    $\mathcal{C}^\infty$-diffeom.

  * $\forall \varphi$ adm. time trafo we have

$$[\mathcal{F}(u \circ \varphi, w^0)](t) = [\mathcal{F}(u, w^0)](\varphi(t))$$

- $\mathcal{F}$, w only depend on range u
- on the order in which these values are attained.

$\rightsquigarrow$ (u,w)-plane



Examples

- superposition operator
- Bouc model $\qquad \varphi'(t) \neq 0$

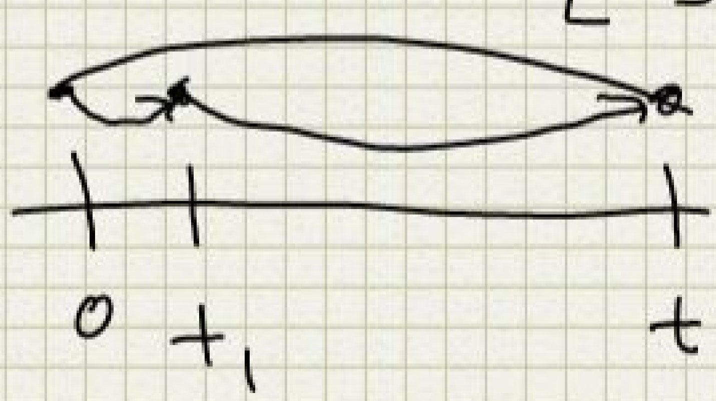
$$(Bu)(t) = c\,u(t) + \int_0^t F\left(\int_s^t |u'(\tau)|\,d\tau\right) \cdot \phi(u(s))\, u'(s)\,ds$$

* Semigroup property

$\forall (u, w^0) \in \mathrm{Dom}(\tilde{F})$, $t_1 < t_2 \in [0,T]$

we have $w(t_1) = [\tilde{F}(u, w^0)](t_1)$

$$w(t_2) = [\tilde{F}(u, w^0)](t_2) = [\tilde{F}(u(t_1 + \cdot), w(t_1))](t_2 - t_1)$$



$$e^{\lambda t_2} = e^{\lambda (t_2 - t_1)} e^{\lambda t_1}$$

* Continuity

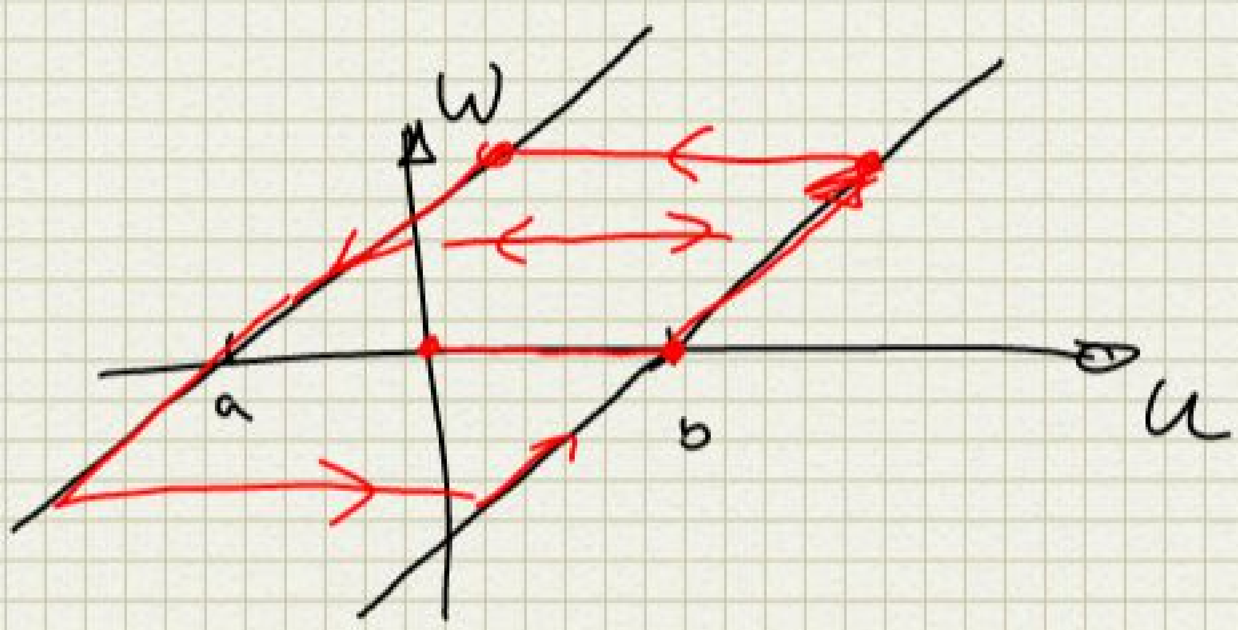
$$\tilde{F} : \mathcal{C}[0,T] \times \mathbb{R} \to \underline{\mathcal{C}[0,T]}$$

$$(u_n, w_n^0) \to (u, w^0)$$

then $\tilde{F}(u_n, w_n^0) \to \tilde{F}(u, w^0)$ unif in time.

Play $\quad K = [a, b] \ni 0$

- $u - w \in K : \quad \dot{w} \cdot (u - w - \xi) \geq 0 \ \forall \xi \in K$
- $\dot{w} \in \underline{\partial I_K (u - w)} = \begin{cases} \emptyset & u - w \notin K \\ 0 & u - w \in \mathring{K} \\ \mathbb{R}_- & u - w = a \\ \mathbb{R}_+ & u - w = b \end{cases}$



Existence

$\forall u \in H^1(0, T, \mathbb{R}) \ \forall w^0 \in \mathbb{R}$

$\exists ! \ w \in H^1(0, T, \mathbb{R})$

Proof: · time discretizat.
· A-priori estimate
· pass to the limit

$\partial_t w_d + \partial_t y - \Delta y = u$ /