P 18 Parameter identification and PML techniques in high-intensity focused ultrasound (HIFU) (B. Wohlmuth, K. Fellner, O. Steinbach) → AO, NS

The aim of this project is to analyze, develop, and implement efficient methods for estimating material parameters in high-intensity focused ultrasound (HIFU) applications. We will consider the common models for nonlinear ultrasound propagation, such as the Westervelt or the more general Kuznetsov equation, which are both quasilinear, strongly damped, possibly degenerate wave equations with nonlinearities of second order. An important task in numerical simulation of waves is to reduce spurious wave reflections off the computational domain. In the present project, we will investigate techniques based on creating an absorbing layer around the computational region.

State of the art. In recent years, high-intensity focused ultrasound (HIFU) has become a reliable medical tool in the noninvasive ablation of tumors in many organs, including the breast [11], liver [5], pancreas [4], and uterus [7]. The efficiency of HIFU in the treatment of tumors is highly dependent on information about tissue parameters, such as the speed of sound, sound diffusivity or the parameter of nonlinearity of a medium. The correct estimate of these parameters would provide a characterization of the state of the tissue and allow to optimize ultrasound focusing.

In [3], inverse methods were developed for identification of acoustic parameters based on modeling the pressure field by the Helmholtz equation. In [10], an adjoint-based parameter estimation approach was considered for a full linear wave equation. Due to the high-pressure amplitudes, nonlinear effects are present in the propagation of HIFU and, for instance, a steepening of the wavefront can be observed. Such behavior cannot be adequately captured by linear wave models. It is therefore essential to take models of nonlinear acoustics into consideration as well, such as the Westervelt or the more general Kuznetsov equation [6]. Recently, in [9], the task of efficient ultrasound focusing was formulated as a shape optimization problem and an adjoint-based, gradient-descent algorithm was developed, where the pressure field was modeled by the Westervelt equation.

In numerical simulation of waves, regions of interest often have to be truncated to a computational domain. A delicate numerical task is to ensure that waves do not falsely reflect off the computational boundary. Perfectly Matched Layer (PML) techniques are based on creating an absorbing layer around the domain for this purpose [2]. By now these techniques have been extended to accommodate various nonlinear wave models, see e.g. [1].

Thesis project to be supervised by B. Wohlmuth. The research in this thesis will be focused on estimating HIFU parameters from measurements of the acoustic pressure. A common model for nonlinear ultrasound propagation in thermoviscous media which will be considered is the Westervelt equation

$$u_{tt} - c^2 \Delta u - b \Delta u_t = \frac{\beta}{\varrho c^2}(u u_{tt} + u_t^2),$$

given here in terms of the acoustic pressure u. The goal will be to recover one of the acoustic parameters, for instance, the speed of sound c, while assuming that the sound diffusivity b, the mass density ϱ, and the coefficient of nonlinearity β of the medium are known.

The task of parameter estimation will be formulated as a PDE constrained optimization problem by introducing a tracking-type objective functional, subject to a quasilinear acoustic wave equation. Suitable regularization methods will have to be taken into account. We will employ the adjoint approach to obtain sensitivity information of the cost functional. A gradient-based algorithm will be derived and implemented, relying on the general approach from [9] to solve the forward and the backward-in-time adjoint problem.

Creating a well-posed, stable, and easy-to-implement PML layer tailored to the nonlinear ultrasound waves is particularly important. Following the strategy from [1], the derivation of the PML for ultrasound waves will be done by linearizing the wave models around a reference solution, deriving the PML for this linearization, and then bringing back in the nonlinear terms. Special attention has to be dedicated to investigations of the well-posedness and stability of the absorbing layers created in such a way.

Further topics. Coupling the pressure field to the (linear) temperature field would allow to observe thermal changes induced by the ultrasound waves. Previous considerations could then be extended to include identifications of material parameters, for instance the thermal conductivity, from temperature measurements. An extension of the results to higher-order acoustic models, such as the third-order-in-time Jordan-Moore-Gibson-Thompson equation [8], is of interest as well.

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