Title: "Evolution reaction-diffusion systems with positivity and mass control: global existence, singular perturbations, L^∞, L^p, L^1, L^2 approaches"

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In this short course, we will address the question of global existence in time (or blow-up in finite time) for so-called "reaction-diffusion systems", which are mathematical models for evolution phenomena undergoing at the same time spatial diffusion and (bio-)chemical type of reactions. Interest has increased recently for these models, in particular for applications in biology, environment and population dynamics.

Two natural properties appear in most models: the nonnegativity of the solutions is preserved for all time; the total mass of the components is controlled for all time (sometimes even exactly preserved). The fact that the total mass of the components does not blow up in finite time suggests that solutions should exist for all time (mathematically speaking, solutions are actually bounded in L^1 uniformly in time). But, it turns out that the answer is not so simple. In particular, blow up in L^∞ may occur in finite time so that it is necessary to give up looking for bounded classical solutions and rather consider *weak solutions*.

We will recall the main results for the 'good' situations where global existence of bounded classical solutions hold. They are obtained through a rather general L^p-duality strategy. After showing how "incomplete blow up" may occur, we will explain how far the notion of global weak solutions gives a satisfactory answer. This mainly relies on an L^1-approach and on truncation techniques. The question of global existence is not completely understood yet and we will indicate open problems.

While these systems offer a good "L^1-structure", they surprisingly also satisfy an a priori L^2-estimate which turns out to be very robust and useful in many other questions: for instance, for the limit of (bio-)chemical systems where some rate constants tend to infinity. If time permits, these singular perturbations will be discussed as well.