

Dispersive Estimates for Schrödinger Equations and Applications

Gerald Teschl

The fact, that harmonic waves with different frequencies travel with different speeds is known as dispersion. Superposition of such waves leads to destructive interference and hence localized wave packages will spread in time yielding decay of the wave package. Mathematically one is interested in quantifying this dispersive decay and proving that the L^p norm of a solution of a linear evolution equation decays like a given power of time provided the initial condition is in some other L^q . Another way of looking at this fact is to look for some averaged version by proving that the L^p norm of a solution is in some L^r as a function of time again provided the initial condition is in some L_q . These latter types of estimates are known as Strichartz estimates.

In addition to providing insight to the behavior of solutions of the linear equations they also constitute an important ingredient for investigating the Cauchy problem for nonlinear perturbations of these equations or for proving stability of soliton solutions of associated nonlinear equations. In this short course we will have a glance at this fascinating area of analysis by looking at the Schrödinger equation from quantum mechanics and the associated nonlinear Schrödinger equation.