Abstracts of Summer School
"Analysis and Applications of Partial Differential Equations"
September 08 – 11, 2014 at Bildungshaus Mariatrost

Michel Pierre

Title: Evolution reaction-diffusion systems with positivity and mass control: Global existence, Singular perturbations, $L\infty$, Lp, $L1$, $L2$ approaches

Abstract:
In this short course, we will address the question of global existence in time (or blow-up in finite time) for so-called "reaction-diffusion systems", which are mathematical models for evolution phenomena undergoing at the same time spatial diffusion and (bio-)chemical type of reactions. Interest has increased recently for these models, in particular for applications in biology, environment and population dynamics.
Two natural properties appear in most models: the nonnegativity of the solutions is preserved for all time; the total mass of the components is controlled for all time (sometimes even exactly preserved). The fact that the total mass of the components does not blow up in finite time suggests that solutions should exist for all time (mathematically speaking, solutions are actually bounded in L^1 uniformly in time). But, it turns out that the answer is not so simple. In particular, blow up in L^∞ may occur in finite time so that it is necessary to give up looking for bounded classical solutions and rather consider weak solutions.
We will recall the main results for the 'good' situations where global existence of bounded classical solutions hold. They are obtained through a rather general L^p -duality strategy. After showing how "incomplete blow up" may occur, we will explain how far the notion of global weak solutions gives a satisfactory answer. This mainly relies on an L^1 -approach and on truncation techniques. The question of global existence is not completely understood yet and we will indicate open problems.
While these systems offer a good "L^1-structure", they surprisingly also satisfy an a priori L^2 -estimate which turns out to be very robust and useful in many other questions: for instance, for the limit of (bio-)chemical systems where some rate constants tend to infinity. If time permits, these singular perturbations will be discussed as well.

Takashi Suzuki

Title: Lotka-Volterra systems - thermodynamics of multi-component models in biology

Abstract:
I talk on the recent study on multi-component reaction-diffusion systems in theoretical biology. First, I describe several examples provided with thermodynamical structures. They are full system of chemotaxis, Gierer-Meinhardt, FitzHugh-Nagumo, cell polarization, prey-predator, cross-diffution, and virus dynamics. Then I turn to skew-symmetric Lotka-Volterra system. By the method of weak scaling limit, I show the global-in-time compactness of the orbit when the space dimension is two.

Gerald Teschl

Title: Dispersive Estimates for Schrödinger Equations and Applications

Abstract:
The fact, that harmonic waves with different frequencies travel with different speeds is known as dispersion. Superposition of such waves leads to destructive interference and hence localized wave packages will spread in time yielding decay of the wave package. Mathematically one is interested in quantifying this dispersive decay and proving that the L^p norm of a solution of a linear evolution equation decays like a given power of time

provided the initial condition is in some other L^q. Another way of looking at this fact is to look for some averaged version by proving that the L^p norm of a solution is in some L^r as a function of time again provided the initial condition is in some L^q. These latter types of estimates are known as Strichartz estimates.

In addition to providing insight to the behavior of solutions of the linear equations they also constitute an important ingredient for investigating the Cauchy problem for nonlinear perturbations of these equations or for proving stability of soliton solutions of associated nonlinear equations. In this short course we will have a glance at this fascinating area of analysis by looking at the Schrödinger equation from quantum mechanics and the associated nonlinear Schrödinger equation.

Hasnaa Zidani

Title: Optimal control and reachability analysis

Abstract:

Contributed talks:

Katrin Grunert

Title: Stability of solutions of the Camassa–Holm equation

Abstract:
The Camassa–Holm equation

$$u_t - u_{txx} + 3uu_x - 2u_xu_{xx} - uu_{xxx} = 0$$

serves as a model for shallow water and has attracted much attention due to its rich mathematical structure. Among other properties even classical solutions enjoy wave breaking, that means the spatial derivative u_x might become unbounded from below pointwise within finite time, while the solution u itself and its H^1-norm remain bounded. Additionally, energy concentrates on sets of measure zero when wave breaking occurs. Thus one question of interest is how to continue solutions beyond wave breaking in order to obtain global solutions as there are at least two possibilities. On the one hand conservative solutions, where the energy is preserved, and on the other hand dissipative solutions, where the concentrated energy vanishes after wave breaking occurs. In this talk we will first show how to describe conservative solutions, before studying the stability of these solutions by deriving a Lipschitz metric. This talk is based on joint work with H. Holden and X. Raynaud.

Evangelos Latos

Title: Existence and Blow-up of Solutions for Semilinear Filtration Problems

Abstract:
We examine the local existence and uniqueness of solutions to the semi-linear filtration equation, with positive initial data and appropriate boundary conditions. Our main result is the proof of blow-up of solutions. Moreover, we discuss about the existence of solutions for the corresponding steady-state problem. It is found that there exists a critical value, above which the problem has no stationary solution of any kind, while below that critical value there exist classical stationary solutions. Exactly this critical value of the parameter acts as a threshold also for the corresponding parabolic problem between blow-up and global existence.

Ngoc-Doanh Nguyen

Title: Asymptotic Behavior of Microbial Degradation Dynamics in Soil

Abstract:
We study the existence and asymptotic behavior of solutions to a microbial degradation model in soil. The model is a system of parabolic partial differential equations with the homogeneous Dirichlet boundary conditions and initial conditions. The existence and uniqueness of non-negative global mild solutions of this system are proved by using the semigroup method. We also prove that under a certain condition there exists a finite-dimensional global attractor of the semigroup generated by the system, and if this condition is invalid, we show the local ultimate boundedness of the solutions. Some biological comments on the results obtained are also given.

Giovanni Pisante

Title: A quantitative Pólya-Szegő inequality

Abstract:

It is well known that the Dirichlet integral \int_{\R^n}|\nabla u|^p decreases under the Steiner symmetrization about an hyperplane or the Schwarz spherical symmetrization about a point. This is indeed a consequence of a Pólya-Szegő type inequality. Recently several studies has been devoted to the issue of characterizing the extremals in this type of inequalities and this led the research through the natural question of proving quantitative versions. The aim of the talk is to present a stability result for the Pólya-Szegő inequality. In particular we will discuss some natural geometric conditions needed to deal with the fact that, even when the Dirichlet integral of a function *u* and of its symmetral coincide, *u* can be very different from its symmetral.

The talk is based on a recent result obtained in collaboration with M. Barchiesi, N. Fusco and G.M. Capriani

Quoq Bao Tang

Title:

Abstract:

Ariane Trescases

Title: From a microscopic model to a macroscopic model with cross-diffusion in Population Dynamics

Abstract:
We consider a class of reaction-cross diffusion systems naturally arising in Population Dynamics. In those systems, cross diffusion terms appear only in one of the two equations (triangular case). We present results of existence of weak solutions of these systems. These solutions are obtained as the limit of the solutions of a microscopic model where only standard diffusions appear. These results use Lyapounov-like functionals and duality lemmas.

Nicola Zamponi

Title: An entropy-based method for the analysis of cross-diffusion PDEs

Abstract:
A systematic method to prove global existence of bounded weak solutions to cross-diffusion PDEs is presented. The method relies on a transformation of variables determined by the entropy density, which is related to the gradient flow formulation of the system. The method is applied to the analysis of a class of cross-diffusion systems for two population species. For such systems, the global-in-time existence of nonnegative bounded weak solutions is shown.
The diffusivities are assumed to depend linearly on the population densities in such a way that a certain formal gradient-flow structure holds. The main feature of these systems is that the diffusion matrix may be neither symmetric nor positive definite. As a particular case, the population model of Shigesada, Kawasaki, and Teramoto in several space dimensions is considered, and the uniform boundedness of weak solutions to such model under certain conditions on the diffusivities is shown for the first time.