Karl-Franzens-Universität Graz

High-frequency behaviour of numerical waves on non-uniform meshes

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Abstract

In this talk, we describe the propagation properties of the one- and two-dimensional wave equation with variable coefficients semi-discretized in space by finite dierence and P1-finite element schemes on non-uniform meshes obtained as diffeomorphic transformations of uniform ones. In particular, we introduce and give a rigorous meaning to notions like the principal symbol of the discrete wave operator or the corresponding bi-characteristic rays. The main mathematical tool we employ is the discrete Wigner transform, which, in the limit as the mesh size parameter tends to zero, yields a measure propagating along curves which are solutions of a Hamiltonian system. Of course, due to dispersion phenomena, the high frequency dynamics does not coincide with the continuous one. Our analysis holds for sufficiently regular coefficients and diffeomorphic transformations defining the grid.
We also present several numerical simulations that confirm the predicted paths of the space-time projections of the bi-characteristic rays. Based on the theoretical analysis and simulations, we describe some of the pathological phenomena that these rays might exhibit as, for example, their reflection before touching the boundary of the space domain. This leads, in particular, to the failure of the classical properties of boundary observability of continuous waves, arising in control and inverse problems theory.

This presentation is based on a joint work with E. Zuazua (BCAM, Bilbao, Spain).