A Certified Reduced Basis Approach for Parametrized Optimal Control Problems

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The solution of optimal control problems governed by PDEs using classical discretization techniques such as finite elements or finite volumes is computationally very expensive and time-consuming since the PDE must be solved many times. One way of decreasing the computational burden is the surrogate model based approach, where the original high-dimensional model is replaced by its reduced order approximation. However, the solution of the reduced order optimal control problem is suboptimal and reliable error estimation is therefore crucial.

A posteriori error estimates have been proposed previously in [4] and [1], respectively. However, the bounds in [4], although rigorous, require solution of the high-dimensional problem and are thus online-inefficient; whereas the estimates in [1], although efficient, are not rigorous upper bounds for the error.

In this talk, we build upon and extend the results in [3, 2] to linear-quadratic optimal control problems governed by parametrized parabolic PDEs. To this end, we employ the reduced basis method as a surrogate model for the solution of the optimal control problem and develop rigorous *and* efficiently evaluable *a posteriori* error bounds for the optimal control *and* the associated cost functional. Besides serving as a certificate of fidelity for the suboptimal solution, our *a posteriori* error bounds are also a crucial ingredient in generating the reduced basis with greedy algorithms.

References

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