Propagation of Mono-kinetic Measures with Rough Initial Profiles

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The analysis of the global flow defined by the Hamiltonian system

$$\begin{aligned} X_t &= \nabla_\xi H(X_t, \Xi_t)\, X_0(x, \Xi) = x, \\ \Xi_t &= \nabla_x H(X_t, \Xi_t)\, \Xi_0(x, \xi) = \nabla_x U(x). \end{aligned}$$

is a standard tool in the WKB asymptotics.
In this talk it will be interpreted as the propagation of a monokinetic measure: The push forward by the Hamiltonian flow of a measure of the form:

$$\mu(x, \xi) = \rho^{in}(x)\delta_{U(x)}(\xi),$$

evolving according to the Liouville equation:

$$\partial_t \mu + \{H, \mu\} = 0.$$

This approach leads us to an estimate of the number of folds of the Lagrangian Manifold even for rough initial data. We also provide informations on the structure of the push-forward $\rho(t, x)$ measure under the canonical projection of the space $\mathbb{R}_x \times \mathbb{R}_\xi$ on $\mathbb{R}_x$.

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