# Adaptive Discontinuous Galerkin Methods for Advection Dominated Optimal Control Problems

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**Abstract**

Many real-life applications such as the shape optimization of technological devices, the identification of parameters in environmental processes and flow control problems lead to optimization problems governed by systems of convection diffusion partial differential equations (PDEs). When convection dominates diffusion, the solutions of these PDEs typically exhibit layers on small regions where the solution has large gradients. Hence, it requires special numerical techniques, which take into account the structure of the convection. The integration of discretization and optimization is important for the overall efficiency of the solution process. Discontinuous Galerkin (DG) methods became recently as an alternative to the finite difference, finite volume and continuous finite element methods for solving wave dominated problems like convection diffusion equations since they possess higher accuracy.
This talk will focus on analysis and application of DG methods for linear-quadratic convection dominated optimal control problems. Because of the inconsistencies of the standard stabilized methods such as streamline upwind Petrov Galerkin (SUPG) on convection diffusion optimal control problems, the *discretize-then-optimize* and the *optimize-then-discretize* do not commute. However, the upwind symmetric interior penalty Galerkin (SIPG) method leads to the same discrete optimality systems. The other DG methods such as non-symmetric interior penalty Galerkin (NIPG) and incomplete interior penalty Galerkin (IIPG) method also yield the same discrete optimality systems when penalization constant is taken large enough. We will study a posteriori error estimates of the upwind SIPG method for the distributed unconstrained and control constrained optimal control problems. In convection dominated optimal control problems with boundary and/or interior layers, the oscillations are propagated downwind and upwind direction in the interior domain, due the opposite sign of convection terms in state and adjoint equations. Hence, we will use residual based a posteriori error estimators to reduce these oscillations around the boundary and/or interior layers. Finally, theoretical analysis will be confirmed by several numerical examples with and without control constraints.

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