# Multi-bang control of elliptic systems

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Multi-bang control refers to optimal control problems for partial differential equations where a distributed control should only take on values from a discrete set of allowed states, which can be promoted by a combination of $L^2$ and $L^0$-type control costs. Although the resulting functional is nonconvex and lacks weak lower-semicontinuity, application of Fenchel duality yields a formal primal-dual optimality system that admits a unique solution. This solution is in general only suboptimal, but the optimality gap can be characterized and shown to be zero under appropriate conditions. Furthermore, in certain situations it is possible to derive a generalized multi-bang principle, i.e., prove that the control almost everywhere takes on allowed values except on sets where the corresponding state reaches the target. A regularized semismooth Newton method allows the numerical computation of (sub)optimal controls. Numerical examples illustrate the effectiveness of the proposed approach and the behavior of multi-bang controls.