On Potts and inverse Potts functionals

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Signals which are constant over large intervals and which only jump at a few locations arise in a large variety of situations in life sciences (e.g. brain stimuli, cross hybridization of DNA, MALDI imaging). The modeling of such signals by piecewise constant functions naturally leads to the minimization of the *Potts functionals*, which are of the form

$$P_\gamma(u) = \gamma \cdot \|\nabla u\|_0 + d(u, f), \tag{1}$$

where the regularity term $\|\nabla u\|_0$ counts the number of jumps of u and $d(u, f)$ measures the error between the solution u and the data f. Motivated by its robustness to non Gaussian noise, we consider a particular instance of (1), the L^1-Potts functional

$$P_\gamma(u) = \gamma \cdot \|\nabla u\|_0 + \|u - f\|_1, \tag{2}$$

where the fidelity is measured in the L^1-norm.

In this talk, after presenting our discretization framework for (2), we present a novel algorithm which efficiently computes exact solutions to the discretized L^1-Potts functional. Moreover, the L^1-Potts functionals turn out to have very interesting blind deconvolution properties, which are not shared by the L^2-Potts and L^1-TV functionals. Our experiments indicate that the deconvolution properties persist also in the presence of various types of noise including non-gaussian noise.

Finally, we consider (inverse) Potts problems, which amount to minimizing functionals of the form

$$\overline{P}_\gamma(u) = \gamma \cdot \|\nabla u\|_0 + \|Au - f\|_p^p. \tag{3}$$

In order to solve (3), we developed an ADMM based algorithm which works well in practice.