Abstract Prof. M. Focardi

Asymptotic analysis of Ambrosio-Tortorelli energies in linearized elasticity

We provide an approximation result in the sense of Γ-convergence for energies of the form

$$\int_\Omega \mathscr{Q}_1(e(u))\,dx + a\,\mathcal{H}^{n-1}(J_u) + b\int_{J_u} \mathscr{Q}_0^{1/2}([u]\odot\nu_u)\,d\mathcal{H}^{n-1},$$

where $\Omega \subset \mathbb{R}^n$ is a bounded open set with Lipschitz boundary, $\mathscr{Q}_0$ and $\mathscr{Q}_1$ are coercive quadratic forms on $\mathbb{M}^{n\times n}_{sym}$, a, b are positive constants, and u runs in the space of fields $SBD^2(\Omega)$, i.e., it's a special field with bounded deformation such that its symmetric gradient $e(u)$ is square integrable, and its jump set J_u has finite $(n-1)$-Hausdorff measure in $\mathbb{R}^n$.

The approximation is performed by means of Ambrosio-Tortorelli type elliptic regularizations, the prototype example being

$$\int_\Omega \left(v|e(u)|^2 + \frac{(1-v)^2}{\varepsilon} + \gamma\,\varepsilon|\nabla v|^2\right)dx,$$

where $(u,v) \in H^1(\Omega,\mathbb{R}^n)\times H^1(\Omega)$, $\varepsilon \le v \le 1$ and $\gamma > 0$.