Error estimates for Dirichlet control problems in polygonal domains

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We study a control constrained Dirichlet optimal control problem governed by an elliptic equation posed on a domain with polygonal boundary. We admit both the cases of a convex or a non-convex domain.

First, we make a detailed study of the regularity of the solution in the corners. Despite the non-convexity of the domain we are able to prove that the optimal control is a continuous function. We can even precise its regularity by means of usual non-weighted Sobolev spaces $W^{1,p}(\Gamma)$, where $p>2$ depends on the greatest convex angle of the domain.

The regularity of the state and of the adjoint state is also studied, both in the framework of usual Sobolev spaces and of weighted Sobolev spaces.

Finally, we obtain error estimates for solutions of the finite element approximations of the problem. We discretize both control and state by means of continuous piecewise linear functions on quasi-uniform meshes.
The order of convergence will depend on the regularity of the data and on the angles of the domain. We show this dependence in a precise way.

Some numerical experiments have been carried out to illustrate the theoretical estimates.