Design of short defibrillation pulses by time optimal control

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Abstract

The electrical behavior of the cardiac tissue is described by the bidomain equations, a set of semilinear reaction-diffusion equations coupled with a set of ODE describing the cell level (e.g. of Hodgkin–Huxley type). The present work is concerned with time optimal control of the monodomain model, which is a simplification of the bidomain model, with the focus on cardiac arrhythmias. The aim is to determine effective and short defibrillation shocks.

Specifically, we look at a sample part of heart tissue that exhibits an undesired electrical behavior in the form of a reentry wave. By placing two electrode plates onto the heart tissue, the electrical behavior can be influenced by the currents applied to the electrodes. The task is to find short and low-energy pulses, that allow for an effective termination of the reentry wave. The proper choice of the cost functional is crucial.

The time optimal control problem is reformulated as a bilevel optimization problem, where the terminal time is fixed in the lower level problem. The lower level problem itself is solved via second-order methods, in particularly a trust-region Newton method. The algorithmic and numerical solution of the bilevel problem is described in detail. The numerical experiments demonstrate that defibrillation pulses designed by the time optimal control approach influence and terminate reentry phenomena effectively.