Optimal Control of the 3D Evolutionary Navier-Stokes Equations

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The velocity tracking problem for the evolutionary Navier-Stokes equations in 3d is studied. The controls are of distributed type and they are submitted to bound constraints. The classical cost functional is modified in such a manner that a full analysis of the control problem is possible. First and second order necessary and sufficient optimality conditions are proved. A fully-discrete scheme based on discontinuous (in time) Galerkin approach combined with conforming finite element subspaces in space, is proposed and analyzed. Provided that the time and space discretization parameters, τ and h respectively, satisfy $\tau \leq Ch^2$, then L^2 error estimates of order O(h) are proved for the difference between the locally optimal controls and their discrete approximations.