Measure valued optimal control problems governed by the linear wave equation

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In this talk we consider optimal control problems governed by the scalar linear wave equation. Our main interest is focused on controls which are measure valued in space and $L^2(0,T)$-functions in time, e.g.,

$u(x,t) = u(t)\delta_{\hat{x}(t)}(x)$ or more general $u(x,t) = u(t)\mu_t(x)$ with $u \in L^2(I)$ and $\mu_t \in \mathcal{M}(\Omega_c)$.

We study two different control spaces, namely the space of L^2-functions in time with values in the space of Radon measures $L^2(0,T;\mathcal{M}(\Omega_c))$ and the space of vector measures $\mathcal{M}(\Omega_c; L^2(0,T))$ with values in $L^2(0,T)$. The major difference between the two approaches is that the first one allows for a time dependent measure part of u, e.g., a moving point source, whereas the second only allows for a time independent measure part. This property can be used to derive higher regularity of the state than in the case of $L^2(0,T;\mathcal{M}(\Omega_c))$. The derivation of the mentioned higher regularity of the state in the case of $\mathcal{M}(\Omega_c; L^2(0,T))$ will be a main part of the talk. Furthermore we study the existence of optimal controls and derive first order optimality conditions for both approaches. Both problem formulations are not amenable to semi-smooth Newton-methods on the continuous level. Therefore we study a L^2-regularized version of the measure valued optimal control problem which then contains the $L^1(\Omega_c; L^2(0,T))$-norm respectively the squared $L^2(0,T;L^1(\Omega_c))$-norm. The semi-smooth Newton-method for optimal control problems involving the $L^1(\Omega_c; L^2(0,T))$-norm is well understood, whereas the case of $L^2(0,T;L^1(\Omega_c))$ is not well understood yet. We will discuss some of the problems. The talk will be concluded with an application in geophysical sciences, namely an inverse seismic source problem. We are interested in the reconstruction of the location and intensity of a seismic event, e.g., an earthquake or a volcano eruption from noisy measurements of the emitted wave. The seismic event will modeled by a fixed point source with time dependent intensity. We confirm the practicability of our approach for such problems in a simplified setting.