Sharp and optimal error estimates for finite element methods of solving parabolic problems in the classes of non-smooth data

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Error estimates are presented for two-level finite element methods of solving parabolic initial-boundary value problems for the Lebesgue or Sobolev-like spaces of right-hand sides and initial data. The estimates (but not their derivation) are rather close to the elliptic case and are given in L^2- and the energy norm. No any a priori assumptions are imposed on solutions. The corresponding lower error estimates are also proved to justify optimality of methods. The estimates are suitable for exploiting in optimal control problems.

The much more difficult case of alternating direction implicit FEMs is also covered. Their error estimates are derived as well, and their sharpness and optimality is also rigorously analyzed in dependence with construction of the methods, the number of space variables and the choice of the data space.