Optimal Control of Free Boundary Problems with Surface Tension Effect

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We will consider a PDE constrained optimization problem governed by a free boundary problem. The state system is based on coupling the Laplace equation in the bulk with a Young-Laplace equation on the free boundary to account for surface tension, as proposed by P. Saavedra and L. R. Scott. This amounts to solving a second order system both in the bulk and on the interface. Our analysis hinges on a convex constraint on the control such that the state constraints are always satisfied. Using only first order regularity we show that the control to state operator is twice Fréchet differentiable. We improve slightly the regularity of the state variables and exploit this to show existence of a control together with second order sufficient optimality conditions.

We prove that the state and adjoint system have the requisite regularity for the error analysis (strong solutions). We discretize the state, adjoint and control variables via piecewise linear finite elements and show optimal *O(h)* error estimates for all variables, including the control.

Finally, we give a novel analysis for a more practical model with Stokes equations in the bulk and slip boundary conditions on the free boundary.