Sparse optimal control of the KDV equation

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We focus in this work on optimal control problems of the following form

$$\min_{q\in\mathcal{M}(\Omega,L^2(I))} J(y) = \frac{1}{2}\|y-y_d\|^2_{L^2(\Omega,L^2(I))} + \alpha\|q\|_{\mathcal{M}_I} \tag{1}$$

where y is the solution of the nonlinear Burgers-Korteweg de Vries equation with a time dependent measure valued source term acting as control

$$\begin{cases} \partial_t y + \partial_x y + \partial_{xxx} y - \gamma\partial_{xx} y + y\partial_x y = q \text{ in } \Omega, \\ y(.,0) = y(.,L) = \partial_x y(.,L) = 0 \text{ in } \Gamma, \\ y(0,.) = 0 \text{ on } \Omega. \end{cases} \tag{2}$$

which is known to have traveling wave solutions. The control space $\mathcal{M}_I$ is the space of vector measures $\mathcal{M}(\Omega.L^2(I))$ with values in $L^2(I)$. For this choice the controls are sparse in space and distributed in time. We will tackle the following theoretical questions: well posedness of the KdV equation with a non-smooth source term, existence and characterization of an optimal control, algorithmic treatment of the problem by a semi-smooth Newton method in function space. In the end, we present some numerical examples that motivate our work: sparse stabilization of the KDV equation and sparse inverse source problems for the KDV equation (reconstruction of the topography and/or topography changes).

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