High Order Quasi Monte-Carlo Integration for countably-parametric operator equations

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We discuss Petrov-Galerkin (PG) discretiations of well-posed, countably-parametric operator equations. Such equations arise commonly from elliptic or parabolic operator equations with distributed uncertainties, such as uncertain coefficient functions, uncertain forcing functions, or uncertain domains. Regularity and sparsity results for the countably-parametric solution families are presented. They are based on analytic continuation of the parametric solutions, in suitable scales of smoothness spaces.

A new class of High-Order, Quasi Monte-Carlo quadrature approximations are presented which afford dimension-independent convergence rates for functionals of the parametric solutions over the (infinite-dimensional) parameter space. The rates are limited only by the sparsity of the parametric solutions, quantified by summability of their polynomial chaos expansions. Numerical results with O(10^3) parameters that confirm the theoretical estimates are presented. Corresponding results for Bayesian estimation are also presented.

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