FE APPROXIMATIONS FOR A FRACTIONAL LAPLACE EQUATION

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Abstract. In this talk we focus on a direct finite element approximation for the Dirichlet homogeneous problem of the so called *integral* fractional Laplacian. Namely, we deal with basic analytical aspects required to convey an "almost" complete Finite Element analysis of the problem

$$\begin{cases} (-\Delta)^s u = f \text{ in } \Omega, \\ u = 0 \text{ in } \Omega^c, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain and the fractional Laplacian of order s is defined by

$$(-\Delta)^s u(x) = C(n,s) \text{ P.V.} \int_{\mathbb{R}^n} \frac{u(x) - u(y)}{|x-y|^{n+2s}} dy,$$

being $C(n,s)$ a normalization constant.

Independently of the Sobolev regularity of the source f, solutions of (1) are not expected to be in a better space than $H^{s+\min\{s,1/2-\epsilon\}}(\Omega)$ (see [3, 5]). However, exploiting Hölder estimates developed in [4], we describe how to obtain further regularity results in a novel framework of weighted fractional Sobolev spaces, leading to a priori estimates in terms of the Hölder regularity of the data [2].

After developing a suitable polynomial interpolation theory in these weighted fractional spaces, optimal order of convergence in the energy norm for the standard linear finite element method is proved for graded meshes. We show some numerical experiments which are in full agreement with our theoretical predictions, and illustrate the optimality of the aforementioned estimates. We also devote some words to discuss basic aspects of the implementation code [1].

Keywords: Fractional Laplacian, Finite Elements, Weighted Fractional Norms, Graded Meshes

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