Algebraic error in numerical PDEs and its estimation

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Estimation of the algebraic error in numerical PDEs presents a very broad topic. Historically, most a posteriori analysis in numerical PDEs focuses on estimating the discretization error and is based on the assumption of the exact solution of the discretized problem. This assumption is certainly restrictive. The fact that the numerical computation is, in general, not accurate, and it is even desirable in many cases not to perform it to a high accuracy, has several consequences that has to be taken into account in practical computations.

This presentation first illustrates, using simple model problems, that the algebraic error can have large local components, which can significantly dominate the total error in some parts of the discretization domain. This can happen despite the fact that the norm of the algebraic error is small in comparison to the norm of the discretization error, and motivates developing a posteriori error estimators that provide the information not only about the global norms but also about the local distribution of the errors.

Using the so-called residual-based error estimator as an example, we study the impact of abandoning the assumption of the exact algebraic solution. In order to allow the evaluation of the estimator at the presence of the algebraic error, the construction of the estimator has to be carefully revisited.

Finally, we show a methodology for computing upper and lower bounds for the algebraic and total error norms based on the flux reconstruction. The derived bounds also allow for estimating the local distribution of the errors over the computational domain. We will discuss bounds on the discretization error, application of the results for constructing mathematically justified stopping criteria for iterative algebraic solvers, and the relationship to the previously published estimates on the algebraic error. The presented results demonstrate the difficulties one has to cope with in rigorous approach for including algebraic errors into the a posteriori estimates. There is also a nonnegligible price in terms of the additional computational cost.

References:

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