A stabilized DG cut cell method for discretizing the linear advection equation

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In recent years, the usage of embedded boundary or cut cell meshes has become increasingly popular. They are an alternative to body-fitted meshes, which may be harder to generate and more complex in the bulk of the flowfield. Cut cell methods cut the flow body out of an structured background grid. This creates so called *cut cells* along the boundary of the embedded object, which have irregular shape and may become very small. These cells need special treatment. For the solution of hyperbolic conservation laws, a major issue caused by cut cells is the small cell problem: standard explicit time stepping schemes are not stable on the arbitrarily small cut cells if the time step is chosen according to the background mesh and does not respect the size of small cut cells.

In this talk we present a new stabilization for overcoming the small cell problem in the context of piecewise linear DG schemes in one and two dimensions for the linear advection equation [1]. Our stabilization is designed to only let a certain portion of the inflow of a small cut cell stay in that small cut cell and to transport the remaining portion directly into the cut cell's outflow neighbors. As a by-product, we reconstruct the proper domain of dependence of the small cut cell's outflow neighbors. In that sense our stabilization relies on similar ideas as the h-box method [2] but without an explicit geometry reconstruction. The approach for realizing these ideas in a DG setting was inspired by the ghost penalty method [3] but significant changes were necessary to adjust the terms that were developed for elliptic problems to the setting of hyperbolic equations.

Using the proposed stabilization, one can use explicit time stepping even on cut cells. In one dimension the stabilized scheme can be shown to be monotone for piecewise constant polynomials and total variation diminishing in the means for piecewise linear polynomials in combination with explicit time stepping schemes. We conclude our talk with numerical results in one and two dimensions.

References

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